

1. Calculate beta ( $\beta$ ) for the solar wind:

$$n = 10 \text{ cm}^{-3} = 10^7 \text{ m}^{-3};$$

$$m = 1.67 \times 10^{-27} \text{ kg};$$

$$v = 400 \text{ km/s} = 4 \times 10^5 \text{ m/s};$$

$$B = 7 \text{ nT} = 7 \times 10^{-9} \text{ Tesla.}$$

$$\beta = \frac{n \left( \frac{1}{2} m v^2 \right)}{B^2 / 2\mu_0} = \frac{10^7 \cdot 0.5 \cdot 1.67 \times 10^{-27} \cdot 16 \times 10^{10}}{49 \times 10^{-18} / 8\pi \times 10^{-7}} = \frac{1.34 \times 10^{-9}}{1.95 \times 10^{-11}} = 68.5$$

- the plasma pressure dominates

2. We need to know the temperature of the stellar plasma before we can answer this question.

3.  $\tan \Psi = \frac{\omega r}{v}$  is the formula. For 400 km/s solar wind, the RHS equals 1, so  $\Psi = 45^\circ$ . Double  $v$ ,  $\tan \Psi$  drops to 0.5,  $\Psi$  drops to  $26.6^\circ$ . This can happen.

4. mostly a matter of opinion, I suppose. Not enough information to really say.

5. Average kinetic energy is:  $\frac{1}{2} m v_{\text{thermal}}^2 = \frac{3}{2} kT$ ,  $v_{\text{thermal}} = \sqrt{3kT/m}$

ions: $v_{\text{thermal}} = \sqrt{3 \cdot 1.6 \times 10^{-19} \cdot 7 / 1.67 \times 10^{-27}}$ $= \sqrt{2.01 \times 10^9} = 4.5 \times 10^4 \frac{\text{m}}{\text{s}} = 45 \frac{\text{km}}{\text{s}}$	electrons: $v_{\text{thermal}} = \sqrt{3 \cdot 1.6 \times 10^{-19} \cdot 10 / 9.1 \times 10^{-31}}$ $= \sqrt{5.27 \times 10^{12}} = 2.3 \times 10^6 \frac{\text{m}}{\text{s}} = 2300 \frac{\text{km}}{\text{s}}$
Mach Number = $\frac{v_{\text{wind}}}{v_{\text{thermal}}} = \frac{400 \frac{\text{km}}{\text{s}}}{45 \frac{\text{km}}{\text{s}}} = 8.9$	Mach Number = $\frac{v_{\text{wind}}}{v_{\text{thermal}}} = \frac{400 \frac{\text{km}}{\text{s}}}{2300 \frac{\text{km}}{\text{s}}} = 0.17$

### Problem 7.

The assignment solution ought to look something like the following, for the given parameters. The point is that the product of the exponentially decaying barometric relation, times the area of the surface ( $r^2$ ), can produce a velocity which is increasing, and can increase to a Mach number above 1. The second figure shows a family of curves, normalized by the appropriate Mach number for each temperature. The primary limitation in this simple look at the acceleration of the solar wind is the neglect of the effect of the plasma pressure on the expansion.

$$n(r) = n_0 \exp \left[ \frac{G M_\odot m_H}{kT R_\odot} \left( \frac{1}{r} - \frac{1}{r_0} \right) \right]$$

$$m_H = 1.67 \times 10^{-27};$$

$$G = 6.67 \times 10^{-11} \text{ Newton-m}^2/\text{kg}^2$$

$$n_0 = \rho_0 / m_H = \frac{10^{-7} \frac{\text{kg}}{\text{m}^3}}{1.67 \times 10^{-27}} = 6 \times 10^{19} \frac{\text{protons}}{\text{m}^3}; \quad r_0 = 1 \text{ solar radius};$$

$$T \text{ is } 1.0 \times 10^6 - 2.0 \times 10^6;$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

$$M_0 = 1.989 \times 10^{30} \text{ kg};$$

$$R_0 = 6.96 \times 10^8 \text{ m}$$

$$n(r) = n_0 \exp \left[ \frac{23.07}{T(\text{millions of degrees})} \left( \frac{1}{r} - \frac{1}{r_0} \right) \right]$$

Flux through the (Gaussian) surface ( $n \cdot v \cdot \text{Area}$ ) is the conserved quantity:

$$n_0 \cdot v_0 \cdot r_0^2 = n \cdot v \cdot r^2. \text{ Taking } r_0 = 1$$

$$n_0 v_0 = n_0 \exp \left[ \frac{G M_0 m_H}{k T R_0} \left( \frac{1}{r} - \frac{1}{r_0} \right) \right] v r^2$$

$$\frac{v}{v_0} = \frac{1}{r^2} \exp \left[ \frac{23.07}{T} \left( 1 - \frac{1}{r} \right) \right]$$

